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A coupled hygro-thermo-mechanical formulation for the analysis of diffusive phenomena in photovoltaic modules

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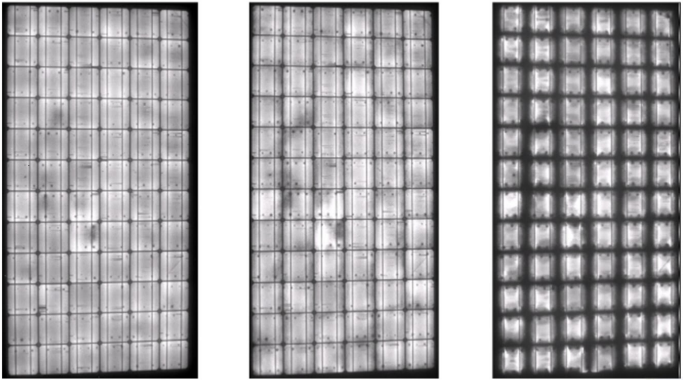
Introduction

EL degradation during the damp heat test ($T = 85^\circ$, $RH = 85\%$) after 1000 h, 2000 h, and 3000 h

[W. Hermann, N. Bogdanski. Outdoor weathering of PV modules–Effects of various climates and comparison with accelerated laboratory laboratory testing, 37th PVSC (IEEE, Seattle, USA, 2011) 2305-2311]



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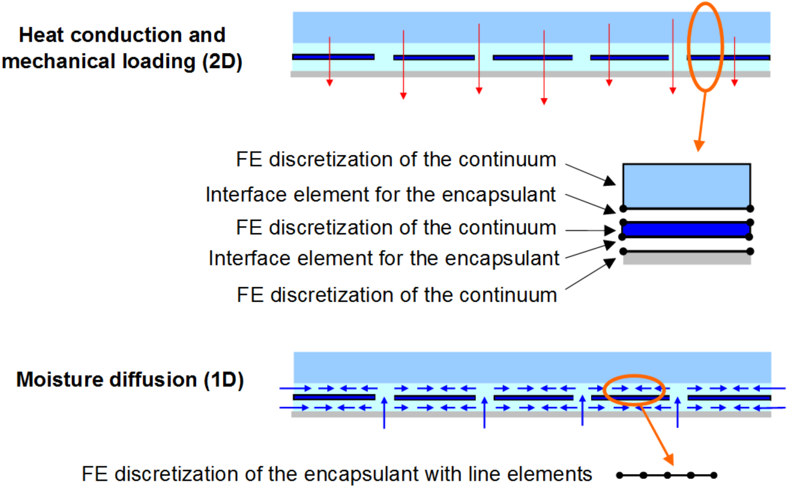
Modelling ideas



Coupling: thermal and mechanical fields influence moisture

Variational framework for the thermo-mechanical behaviour of EVA

Interface contribution to the Principle of Virtual Work:

$$\Pi_{\mathrm{m}} = \int_{S_0} \mathbf{g}_{\mathrm{loc}}^{T} \mathbf{t} \mathrm{d}S,$$

where $\mathbf{g}_{\mathrm{loc}}$ and $\mathbf{t}$ denote the relative displacement vector and the cohesive traction vector, respectively

Interface contribution to the energy balance for heat conduction:

$$\Pi_{\mathrm{th}} = \int_{S_0} g_{\mathrm{th}} q \mathrm{d}S,$$

where g_{th} and q denote the relative temperature between the interface sides and the heat flux, respectively

Variational framework for the thermo-mechanical behaviour of EVA

Virtual variation of Π_{m} w.r.t. the displacement field:

$$\delta\Pi_{\mathrm{m}} = \delta\mathbf{u}^{\mathrm{T}} \int_{S_0} \left(\frac{\partial \mathbf{g}_{\mathrm{loc}}}{\partial \mathbf{u}}\right)^{\mathrm{T}} \mathbf{t}\,\mathrm{d}S$$

Virtual variation of Π_{th} w.r.t. the temperature:

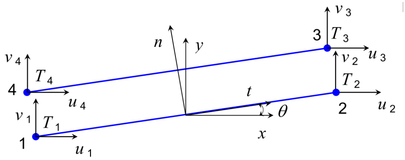
$$\delta\Pi_{\mathrm{th}} = \delta T \int_{S_0} \left(\frac{\partial g_{\mathrm{th}}}{\partial T}\right) q\,\mathrm{d}S$$

FE approximation

After introducing the FE discretization, we consider the middle line (in 2D) or the surface (in 3D) of the interface element defined by a local frame with a rotation matrix **R** whose components are related to the following vectors:

$$\mathbf{t_1} = \frac{\partial \overline{\mathbf{x}}^e}{\partial \xi}, \quad \mathbf{n} \cdot \mathbf{t_1} = 0, \quad \text{in 2D}$$

$$\mathbf{t}_1 = \frac{\partial \overline{\mathbf{x}}^e}{\partial \xi}, \quad \mathbf{t}_2 = \frac{\overline{\mathbf{x}}^e}{\partial \eta}, \quad \mathbf{n} = \mathbf{t}_1 \times \mathbf{t}_2, \quad \text{in 3D}$$



FE approximation

We relate $\mathbf{g}_{\mathrm{loc}}$ and g_{th} to the nodal displacement vector $\mathbf{d}_{\mathrm{m}}$ and to the nodal temperature vector $\mathbf{d}_{\mathrm{th}}$:

$$\mathbf{g}_{\mathrm{loc}} \cong \mathbf{g}^{e}_{\mathrm{loc}} = \mathbf{R}\mathbf{N}_{\mathrm{m}}\mathbf{L}_{\mathrm{m}}\mathbf{d}_{\mathrm{m}}$$

$$g_{\mathrm{th}} \cong g^{e}_{\mathrm{th}} = \mathbf{N}_{\mathrm{th}}\mathbf{L}_{\mathrm{th}}\mathbf{d}_{\mathrm{th}}$$

FE approximation

Virtual variations of Π_{m} and Π_{th} in the FE approximation:

$$\delta\Pi_{\mathrm{m}} \sim \delta\mathbf{d}_{\mathrm{m}}^{\mathrm{T}} \int_{S_0} \left(\frac{\partial \mathbf{g}_{\mathrm{loc}}^{e}}{\partial \mathbf{d}_{\mathrm{m}}}\right)^{\mathrm{T}} \mathbf{t}\,\mathrm{d}S$$

$$\delta\Pi_{\mathrm{th}} \sim \delta\mathbf{d}_{\mathrm{th}}^{\mathrm{T}} \int_{S_0} \left(\frac{\partial g_{\mathrm{th}}^{e}}{\partial \mathbf{d}_{\mathrm{th}}}\right)^{\mathrm{T}} q\,\mathrm{d}S$$

where:

$$\frac{\partial \mathbf{g}_{\mathrm{loc}}^{e}}{\partial \mathbf{d}_{\mathrm{m}}} = \mathbf{R}\mathbf{N}_{\mathrm{m}}\mathbf{L}_{\mathrm{m}} = \mathbf{R}\mathbf{B}_{\mathrm{m}}$$

$$\frac{\partial g_{\mathrm{th}}^{e}}{\partial \mathbf{d}_{\mathrm{th}}} = \mathbf{N}_{\mathrm{th}}\mathbf{L}_{\mathrm{th}} = \mathbf{B}_{\mathrm{th}}$$

Residual vector

Discretized version of the virtual variation of Π_{m}^{e}:

$$\delta\Pi_{\mathrm{m}}^{e} = \delta\mathbf{d}_{\mathrm{m}}^{\mathrm{T}} \int_{S_0} (\mathbf{R}\mathbf{B}_{\mathrm{m}})^{\mathrm{T}}\,\mathbf{t}\,\mathrm{d}S$$

Discretized version of the virtual variation of Π_{th}^{e}:

$$\delta\Pi_{\mathrm{th}}^{e} = \delta\mathbf{d}_{\mathrm{th}}^{\mathrm{T}} \int_{S_0} \mathbf{B}_{\mathrm{th}}{}^{\mathrm{T}} q\,\mathrm{d}S$$

The solution of $\delta\Pi_{\mathrm{m}}^{e} = \delta\mathbf{d}_{\mathrm{m}}^{\mathrm{T}}\mathbf{f}_{\mathrm{m}}^{e} = 0\ \forall\,\delta\mathbf{d}_{\mathrm{m}}$ and $\delta\Pi_{\mathrm{th}}^{e} = \delta\mathbf{d}_{\mathrm{th}}^{\mathrm{T}}\mathbf{f}_{\mathrm{th}}^{e} = 0$ $\forall\,\delta\mathbf{d}_{\mathrm{th}}$ provides the components of the residual vector $\mathbf{f}^{e} = (\mathbf{f}_{\mathrm{m}}^{e}, \mathbf{f}_{\mathrm{th}}^{e})$:

$$\mathbf{f}_{\mathrm{m}}^{e} = \int_{S_0} (\mathbf{R}\mathbf{B}_{\mathrm{m}})^{\mathrm{T}}\,\mathbf{t}\,\mathrm{d}S$$

$$\mathbf{f}_{\mathrm{th}}^{e} = \int_{S_0} \mathbf{B}_{\mathrm{th}}{}^{\mathrm{T}} q\,\mathrm{d}S$$

Tangent stiffness matrix

The linearization of the residual vector provides the tangent stiffness matrix components:

$$\mathbf{K}^{e,k}_{\mathrm{m,m}} = \frac{\partial \mathbf{f}^{e,k}_{\mathrm{m}}}{\partial \mathbf{d}_{\mathrm{m}}} = \int_{S_0} \mathbf{B}^{\mathrm{T}}_{\mathrm{m}} \mathbf{R}^{\mathrm{T}} \frac{\partial \mathbf{t}}{\partial \mathbf{d}} \,\mathrm{d}S = \int_{S_0} \mathbf{B}^{\mathrm{T}}_{\mathrm{m}} \mathbf{R}^{\mathrm{T}} \mathbf{C}_{\mathrm{m,m}} \mathbf{R} \mathbf{B}_{\mathrm{m}} \,\mathrm{d}S$$

$$\mathbf{K}^{e,k}_{\mathrm{m,th}} = \frac{\partial \mathbf{f}^{e,k}_{\mathrm{m}}}{\partial \mathbf{d}_{\mathrm{th}}} = \int_{S_0} \mathbf{B}^{\mathrm{T}}_{\mathrm{m}} \mathbf{R}^{\mathrm{T}} \frac{\partial \mathbf{t}}{\partial \mathbf{d}_{\mathrm{th}}} \,\mathrm{d}S = \int_{S_0} \mathbf{B}^{\mathrm{T}}_{\mathrm{m}} \mathbf{R}^{\mathrm{T}} \mathbf{C}_{\mathrm{m,th}} \,\mathrm{d}S$$

$$\mathbf{K}^{e,k}_{\mathrm{th,m}} = \frac{\partial \mathbf{f}^{e,k}_{\mathrm{th}}}{\partial \mathbf{d}_{\mathrm{m}}} = \int_{S_0} \mathbf{B}^{\mathrm{T}}_{\mathrm{th}} \frac{\partial q}{\partial \mathbf{d}_{\mathrm{m}}} \,\mathrm{d}S = \int_{S_0} \mathbf{B}^{\mathrm{T}}_{\mathrm{th}} \frac{\partial q}{\partial g_{\mathrm{n}}} \mathbf{e}_{\mathrm{n}} \mathbf{B}_{\mathrm{m}} \,\mathrm{d}S$$

$$\mathbf{K}^{e,k}_{\mathrm{th,th}} = \frac{\partial \mathbf{f}^{e,k}_{\mathrm{th}}}{\partial \mathbf{d}_{\mathrm{th}}} = \int_{S_0} \mathbf{B}^{\mathrm{T}}_{\mathrm{th}} \frac{\partial q}{\partial \mathbf{d}_{\mathrm{th}}} \,\mathrm{d}S = \int_{S_0} \mathbf{B}^{\mathrm{T}}_{\mathrm{th}} \mathbf{C}_{\mathrm{th,th}} \mathbf{B}_{\mathrm{th}} \,\mathrm{d}S$$

Newton-Raphson scheme

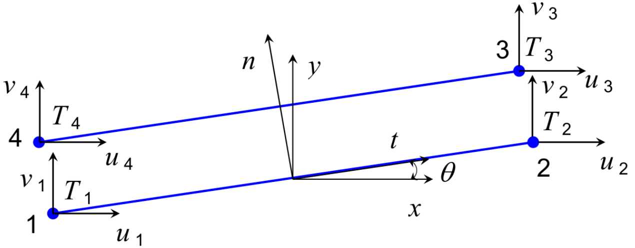
The following equations set for the computation of the corrector $\Delta\mathbf{d} = (\Delta\mathbf{d}_{\mathrm{m}}, \Delta\mathbf{d}_{\mathrm{th}})^{\mathrm{T}}$ at each iteration k is used:

$$\mathbf{K}^{e,k}\Delta\mathbf{d} = -\mathbf{f}^{e,k}$$
$$\mathbf{d}^{k+1} = \mathbf{d}^{k} + \Delta\mathbf{d}$$

where:

$$\mathbf{K}^{e,k} = \begin{bmatrix} \mathbf{K}^{e,k}_{\mathrm{m,m}} & \mathbf{K}^{e,k}_{\mathrm{m,th}} \\ \mathbf{K}^{e,k}_{\mathrm{th,m}} & \mathbf{K}^{e,k}_{\mathrm{th,th}} \end{bmatrix}$$

2D interface element



$$\mathbf{d}_{\mathrm{m}} = (u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4)^T$$

$$\mathbf{d}_{\mathrm{th}} = (T_1, T_2, T_3, T_4)^T$$

Matrix operators: mechanical part

The matrix operators for the mechanical part are:

$$\mathbf{N}_{\mathrm{m}} = [N_1\mathbf{I} \;\; N_2\mathbf{I}]$$

where $N_1 = (1-\xi)/2$ and $N_2 = (1+\xi)/2$, $\xi \in [-1,+1]$

$$\mathbf{L}_{\mathrm{m}} = \begin{bmatrix} -\mathbf{I} & \mathbf{O} & \mathbf{O} & \mathbf{I} \\ \mathbf{O} & -\mathbf{I} & \mathbf{I} & \mathbf{O} \end{bmatrix}$$

with $\mathbf{I}$ and $\mathbf{O}$ 2×2 identity and zero matrices

$$\mathbf{R} = \begin{bmatrix} t_{1,x} & t_{1,y} \\ n_x & n_y \end{bmatrix}$$

Matrix operators: mechanical part

The traction vector in 2D reads:

$$\mathbf{t} = (\tau, \sigma)^T$$

and the constitutive equation (linear tension cut-off cohesive zone model) is:

$$\mathbf{t} = \mathbf{C}_{\mathrm{m,m}} \mathbf{g}_{\mathrm{loc}}$$

where:

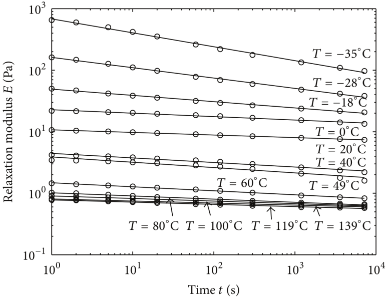
$$\mathbf{C}_{\mathrm{m,m}} = \frac{\partial \mathbf{t}}{\partial \mathbf{g}_{\mathrm{loc}}} = \begin{bmatrix} G/h & 0 \\ 0 & E/h \end{bmatrix}$$

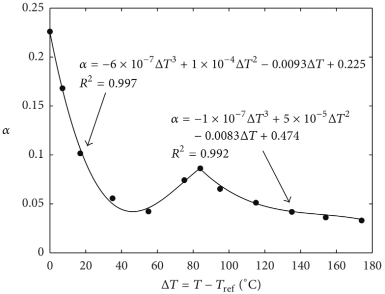
Assuming that $E = E(\overline{T})$ and $G = G(\overline{T})$, where $\overline{T} = \mathbf{N}_{\mathrm{th}} \mathbf{M}_{\mathrm{th}} \mathbf{d}_{\mathrm{th}}$:

$$\mathbf{C}_{\mathrm{m,th}} = \frac{\partial \mathbf{t}}{\partial \mathbf{d}_{\mathrm{th}}} = \frac{\partial \mathbf{t}}{\partial \overline{T}} \frac{\partial \overline{T}}{\partial \mathbf{d}_{\mathrm{th}}} = \frac{\partial \mathbf{t}}{\partial \overline{T}} \mathbf{N}_{\mathrm{th}} \mathbf{M}_{\mathrm{th}}$$

A thermo-viscoelastic model based on fractional calculus

$$E(t,\overline{T}) = a(\overline{T})\frac{t^{-\alpha(\overline{T})}}{\Gamma(1-\alpha(\overline{T}))}$$





Paggi, M and Sapora, A. An accurate thermo-visco-elastic rheological model for ethylene vinyl acetate based on fractional calculus. Int J Photoenergy, in press, paper 252740

Matrix operators: thermal part

The matrix operators for the thermal part are:

$$\mathbf{N}_{\mathrm{th}} = [N_1 \quad N_2]$$

where $N_1 = (1-\xi)/2$ and $N_2 = (1+\xi)/2$, $\xi \in [-1,+1]$

$$\mathbf{L}_{\mathrm{th}} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

Considering a Fourier type constitutive law for the interface:

$$q = \left(1 - \frac{g_{\mathrm{n}}}{g_{\mathrm{n,c}}}\right) q_0 = -\left(1 - \frac{g_{\mathrm{n}}}{g_{\mathrm{n,c}}}\right) k_{\mathrm{th}} g_{\mathrm{th}} = -\left(1 - \frac{g_{\mathrm{n}}}{g_{\mathrm{n,c}}}\right) k_{\mathrm{th}} \mathbf{B}_{\mathrm{th}} \mathbf{d}_{\mathrm{th}}$$

we obtain:

$$\mathbf{C}_{\mathrm{th,m}} = \frac{\partial q}{\partial \mathbf{d}_{\mathrm{m}}} = -\frac{q_0}{g_{\mathrm{nc}}} \mathbf{e}_{\mathrm{n}} \mathbf{B}_{\mathrm{m}}; \quad \mathbf{C}_{\mathrm{th,th}} = \frac{\partial q}{\partial \mathbf{d}_{\mathrm{th}}} = -k_{\mathrm{th}} \left(1 - \frac{g_{\mathrm{n}}}{g_{\mathrm{n,c}}}\right) \mathbf{B}_{\mathrm{th}}$$

Variational framework for moisture diffusion inside EVA

The mass conservation law governing moisture diffusion is:

$$\rho \frac{\partial c}{\partial t} = -\frac{\partial \eta}{\partial s}$$

where η is the flux, $c(s,t)$ is the concentration, and ρ is the EVA mass density. We assume the Fick's law holds:

$$\eta = -D \frac{\partial c}{\partial s}$$

where $D = D(\overline{T}, g_{\mathrm{n}})$ is the diffusion coefficient.
The PDE governing moisture diffusion is:

$$\rho \frac{\partial c}{\partial t}(s,t) - D \frac{\partial^2 c}{\partial s^2}(s,t) = 0$$

The weak form of the problem reads:

$$G(c, \delta c) = \int_\Gamma \frac{\partial c}{\partial t} \delta c \mathrm{d}s + \int_\Gamma \frac{\partial c}{\partial s} D \frac{\partial \delta c}{\partial s} \mathrm{d}s = 0$$

FE discretization in space

The isoparametric FE discretization with linear shape functions $N_1 = (1 - \xi)/2$ and $N_2 = (1 + \xi)/2$ is introduced:

$$s = \mathbf{N}_c \mathbf{x}, \quad c = \mathbf{N}_c \mathbf{c}, \quad \delta c = \mathbf{N}_c \delta \mathbf{c}$$

and, within the Bubnov-Galerkin framework, the semi-discretized weak form reads:

$$G_h = \delta \mathbf{c}^{\mathrm{T}} \left(\int_{\Gamma_h} \rho \mathbf{N}_c^{\mathrm{T}} \mathbf{N}_c \, \mathrm{d}s \right) \dot{\mathbf{c}} + \delta \mathbf{c}^{\mathrm{T}} \left(\int_{\Gamma_h} D \mathbf{B}_{\mathrm{c}}^{\mathrm{T}} \mathbf{B}_{\mathrm{c}} \, \mathrm{d}s \right) \mathbf{c} = 0, \ \forall \delta \mathbf{c} \in \mathbb{R}^{2\times 1}$$

Leading to the following matrix form:

$$\mathbf{M}\dot{\mathbf{c}}(t) + \mathbf{D}\mathbf{c}(t) = \mathbf{0}$$

where $\mathbf{M} = \int_{\Gamma_h} \rho \mathbf{N}_c^{\mathrm{T}} \mathbf{N}_c \, \mathrm{d}s, \quad \mathbf{D} = \int_{\Gamma_h} D \mathbf{B}_{\mathrm{c}}^{\mathrm{T}} \mathbf{B}_{\mathrm{c}} \, \mathrm{d}s$

Time integration

The backward Euler method (implicit solution scheme) is adopted:

$$(\mathbf{M} + \mathbf{D}\Delta t)\,\mathbf{c}^{m+1} = \mathbf{M}\mathbf{c}^{m}, \quad m = 1, \dots, M$$

The residual vector is defined as:

$$\mathbf{f}_c^e = \int_{\Gamma_h} \rho \mathbf{N}_c^{\mathrm{T}} \dot{c}\, \mathrm{d}s - \int_{\Gamma_h} \mathbf{B}_{\mathrm{c}}^{\mathrm{T}} \eta\, \mathrm{d}s$$

where:

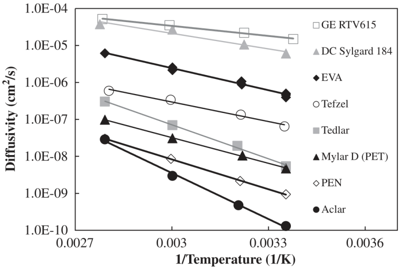
$$\dot{c} = \mathbf{N}_c \dot{\mathbf{c}}^m, \quad \eta = -D\mathbf{B}_{\mathrm{c}}\mathbf{c}^m$$

$$\mathbf{N}_c = [N_1 \ N_2] \quad \mathbf{B}_{\mathrm{c}} = \left[\frac{\partial N_1}{\partial \xi} \ \frac{\partial N_2}{\partial \xi}\right]$$

Dependency of D on $\overline{T}$ and $g_{\rm n}$

The diffusivity depends on $\overline{T}$ according to an Arrhenius equation [M.D. Kempe, Sol. Mat. & Solar Cells 90 (2006) 2720–2738]

$$D_0 = A \exp\left[-E_a/(R\overline{T})\right]$$



We also postulate a linearly dependency on $g_{\rm n}$ as:

$$D = D_0 \ \text{ for } g_n < g_{nc}\,, \quad D = D_0 \frac{g_n}{g_{nc}} \ \text{ for } g_n \geq g_{nc}$$

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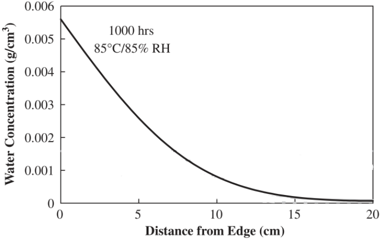
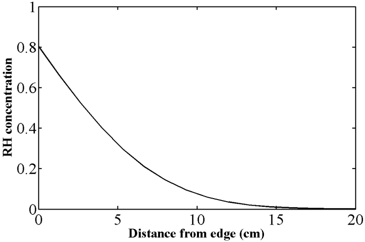
Numerical example (1)

$$D = 5 \times 10^{-5}\ \mathrm{cm}^2/\mathrm{s},\ \overline{T} = 80^\circ\mathrm{C}$$

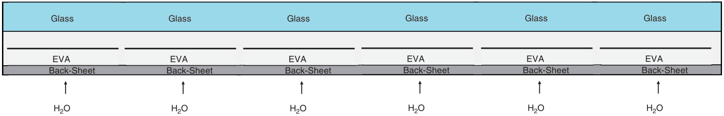


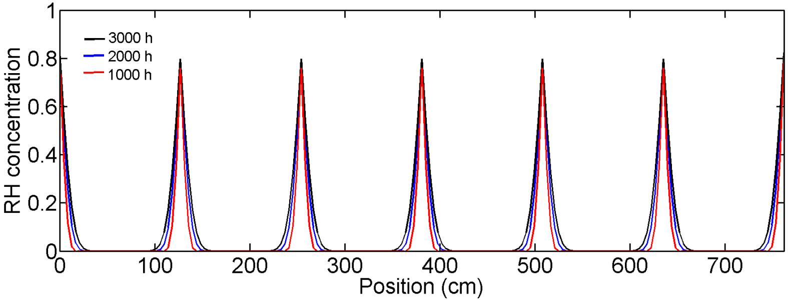
(c) Kempe (2006)



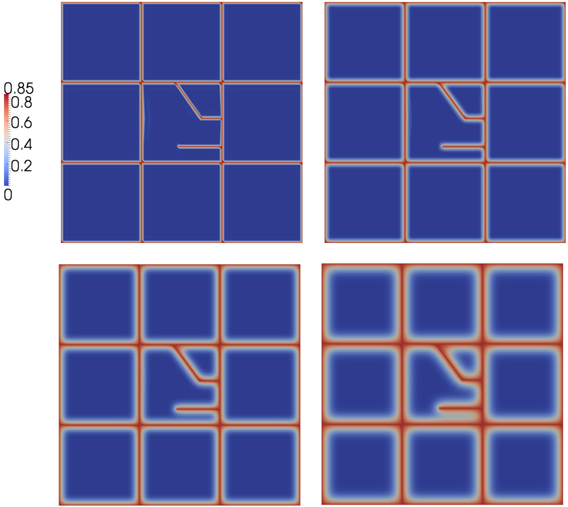
(d) Numerical results

Numerical example (2)





Numerical example (3)



t=1s, 1000h, 3000h, 20000h

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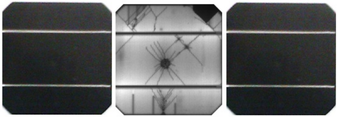
Conclusion and outlook

- Variational framework for studying hygro-thermo-mechanical problems in PV modules
- Tangent operators for implicit solution schemes
- Simplification of the encapsulant with interface elements
- Coupling between moisture diffusion and the other fields accounted for in the constitutive parameters
- Preliminary examples are promising and future developments regard applications to 3D problems and experimental comparisons

Acknowledgements

Multi-field and multi-scale Computational Approach to design and durability of Photovoltaic Modules – CA2PVM



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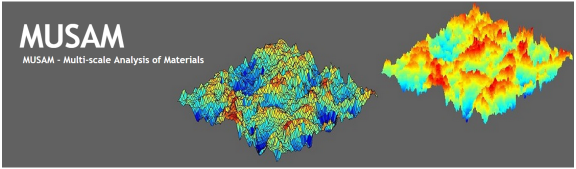
http://musam.imtlucca.it/CA2PVM.html



Annual report 2014: http://musam.imtlucca.it/Report_2014.pdf